Class 12 - Mathematics

Mathematics Part - II

Integrals

CBSE NOTES

Integrals - Mastery Worksheet

Strengthen your foundation with key concepts and basic applications.

Mastery Questions

Solve the following questions. Write your answers in the space provided.

1. Find the area under the curve defined by $f(x) = x^3 - 3x + 2$ between $x = -2$ and $x = 2$, and confirm your result using definite integrals.

Hint: Use the Fundamental Theorem of Calculus and consider the symmetry of the function about the y-axis.

2. Evaluate the integral $\int(3x^2 - 4x + 1)\,dx$ and interpret it geometrically.

Hint: Focus on understanding how the coefficients of the polynomial impact the area.

Mastery Questions

Solve the following questions. Write your answers in the space provided.

3. Using substitution, evaluate ∫(sin(x)cos(x)) dx.

Hint: Identify suitable substitutions that simplify the integrand effectively.

4. Demonstrate the integration by parts method on ∫x * e^x dx.

Hint: Choose u and dv thoughtfully, and remember the formula ∫u dv = u*v - ∫v du.

Mastery Questions

Solve the following questions. Write your answers in the space provided.

5. Solve the definite integral ∫(0 to π) (sin^2(x)) dx using the identity sin^2(x) = (1 - cos(2x))/2.

Hint: Transform the integral using trigonometric identities for easier calculation.

6. Explain and compute ∫(x^2)/(x^3 + 1) dx using partial fractions.

Hint: Partial fractions help break down complex rational functions into simpler integrals.

Solve the following questions. Write your answers in the space provided.

7. Evaluate the integral ∫e^(3x) * cos(2e^(3x)) dx using the substitution method.

Hint: Look for exponentials and trigonometric functions together to create an effective substitution.

8. Determine the integral ∫(x^2 - 4)/(x^2 + 1)^2 dx, using appropriate techniques.

Hint: Firstly, simplify the rational function to spot opportunities for standard integral forms.

Mastery Questions

Solve the following questions. Write your answers in the space provided.

9. Prove that ∫(1/x^2) dx = -1/x + C and discuss its implications for the function on the intervals (0, ∞) and (-∞, 0).

Hint: Consider the function's limits approaching the points of discontinuity when discussing implications.

10. Investigate the integral ∫(x * ln(x)) dx using integration by parts, and conclude by evaluating it.

Hint: Plan your choice of u and dv to reduce the resulting integral effectively.

Answers

Check your answers with the solutions below.

1. Find the area under the curve defined by f(x) = x^3 - 3x + 2 between x = -2 and x = 2, and confirm your result using definite integrals.

Solution: To find the area, first determine the antiderivative F(x) = (1/4)x^4 - (3/2)x^2 + 2x. Evaluating F(2) and F(-2) gives the area: A = |F(2) - F(-2)|. Simplifying gives A = 16 square units.

2. Evaluate the integral ∫(3x^2 - 4x + 1) dx and interpret it geometrically.

Solution: The antiderivative is F(x) = x^3 - 2x^2 + x + C. Geometrically, this represents the change in area as the function is integrated, reflecting the net area under the curve.

3. Using substitution, evaluate ∫(sin(x)cos(x)) dx.

Solution: Using u-substitution with u = sin(x), du = cos(x)dx. This transforms the integral to ∫(u) du = (1/2)u^2 + C = (1/2)sin^2(x) + C.

4. Demonstrate the integration by parts method on ∫x * e^x dx.

Solution: Let u = x, dv = e^x dx. Then, du = dx and v = e^x. Applying integration by parts gives ∫x * e^x dx = x * e^x - ∫e^x dx = x * e^x - e^x + C.

5. Solve the definite integral ∫(0 to π) (sin^2(x)) dx using the identity sin^2(x) = (1 - cos(2x))/2.

Solution: Calculate ∫(0 to π) (1/2 - 1/2cos(2x)) dx = (1/2)x - (1/4)sin(2x) | from 0 to π = (π/2) - 0 = π/2.

Answers

Check your answers with the solutions below.

6. Explain and compute ∫(x^2)/(x^3 + 1) dx using partial fractions.

Solution: Decompose as A/(x + 1) + B/(x^2), solving gives A and B. The integral becomes ∫A/(x + 1) dx + ∫B/(x^2) dx yielding results based on their respective antiderivatives.

7. Evaluate the integral ∫e^(3x) * cos(2e^(3x)) dx using the substitution method.

Solution: Let u = e^(3x), then dx = (1/3)(1/u) du. Substitute into the integral giving ∫cos(2u) * (1/3) du, resulting in (1/3)(1/2)sin(2u)/2 + C.

8. Determine the integral ∫(x^2 - 4)/(x^2 + 1)^2 dx, using appropriate techniques.

Solution: Decompose into simpler fractions, with integrals of the form ∫1/(x^2 + 1)^2 dx yielding well-known antiderivatives involving arctangent.

9. Prove that ∫(1/x^2) dx = -1/x + C and discuss its implications for the function on the intervals (0, ∞) and (-∞, 0).

Solution: Use standard integral rules; the antiderivative indicates the original function. It diverges at x = 0, showing behavior in both positive and negative directions.

10. Investigate the integral ∫(x * ln(x)) dx using integration by parts, and conclude by evaluating it.

Solution: Let u = ln(x) and dv = x dx. This gives du = (1/x)dx and v = (x^2)/2. The solution becomes (x^2/2)ln(x) - ∫(1/2)x dx = (x^2/2)ln(x) - (1/4)x^2 + C.

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