Class 12 - Mathematics

Mathematics Part - I

Determinants

CBSE NOTES

Determinants - Mastery Worksheet

Strengthen your foundation with key concepts and basic applications.

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Mastery Questions

Solve the following questions. Write your answers in the space provided.

1. Given two matrices A = [[2, 3], [1, 4]] and B = [[1, 2], [3, 4]], find the determinants of both matrices and explain what these values indicate about the systems of equations represented by the matrices.

Hint: Use the formula for 2x2 determinants: $|A| = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$.

Mastery Questions

Solve the following questions. Write your answers in the space provided.

2. Illustrate how to find the area of a triangle formed by the vertices (1, 2), (3, 4), and (5, 0) using determinants. Show all steps of your calculation.

Hint: Set up the determinant matrix properly to include the homogeneous coordinate.

3. Prove that if matrix A is multiplied by a scalar k, then its determinant is multiplied by k^n, where n is the order of the matrix.

Hint: Consider the determinant's properties under scalar multiplication.

Mastery Questions

Solve the following questions. Write your answers in the space provided.

4. Compute the determinant of the matrix A = [[1, 2, 3], [4, 5, 6], [7, 8, 9]] and discuss whether it is singular or non-singular. Justify your answer.

Hint: Expand the determinant using the first row.

5. Find the values of x for which the determinant of the matrix B = [[3, 1, 2], [1, x, 4], [2, 5, 3]] is zero.

Hint: Expand the determinant and set it equal to zero.

Solve the following questions. Write your answers in the space provided.

6. Discuss the relationship between the determinants of a matrix and its inverse. What condition must the original matrix satisfy for its inverse to exist?

Hint: Consider properties of inverses in linear algebra.

7. Calculate the determinant of the matrix C = [[sin(x), cos(x)], [cos(x), -sin(x)]] and interpret the result.

Hint: Use trigonometric identities for simplification.

Solve the following questions. Write your answers in the space provided.

8. Using Cramer's Rule, solve the system of equations represented by A = [[2, 3], [1, 2]], bx = [[7], [4]] for x.

Hint: Set up determinants for each variable as per Cramer's rule.

9. Find the minor and cofactor of the element a23 in the matrix D = [[2, 3, 4], [5, 6, 7], [8, 9, 1]].

Hint: Calculate the determinant of the 2x2 matrix formed after deletion of appropriate row and column.

Solve the following questions. Write your answers in the space provided.

10. Explain why the area of a triangle formed by collinear points is zero. Use a determinant to illustrate your explanation.

Hint: Rewrite the determinant format for triangle area.

Answers

Check your answers with the solutions below.

1. Given two matrices A = [[2, 3], [1, 4]] and B = [[1, 2], [3, 4]], find the determinants of both matrices and explain what these values indicate about the systems of equations represented by the matrices.

Solution: Det(A) = (2*4) - (3*1) = 8 - 3 = 5. Det(B) = (1*4) - (2*3) = 4 - 6 = -2. A non-zero determinant indicates that both systems have unique solutions.

2. Illustrate how to find the area of a triangle formed by the vertices (1, 2), (3, 4), and (5, 0) using determinants. Show all steps of your calculation.

Solution: Area = 1/2 * |det([[1, 2, 1], [3, 4, 1], [5, 0, 1]])| = 1/2 * |1(4-0) + 3(0-2) + 5(2-4)| = 1/2 * |4 - 6 - 10| = 1/2 * |-12| = 6.

3. Prove that if matrix A is multiplied by a scalar k, then its determinant is multiplied by k^n, where n is the order of the matrix.

Solution: Let A be an n x n matrix. If A' = kA, then |A'| = k^n |A|. For example, if A is a 2x2 matrix, multiplying A by 2 results in |2A| = 2^2*|A| = 4|A|.

4. Compute the determinant of the matrix A = [[1, 2, 3], [4, 5, 6], [7, 8, 9]] and discuss whether it is singular or non-singular. Justify your answer.

Solution: Det(A) = 1(5*9 - 6*8) - 2(4*9 - 6*7) + 3(4*8 - 5*7) = 1(45 - 48) - 2(36 - 42) + 3(32 - 35) = -3 + 12 - 9 = 0. A singular matrix indicates no unique solution exists.

Check your answers with the solutions below.

5. Find the values of x for which the determinant of the matrix B = [[3, 1, 2], [1, x, 4], [2, 5, 3]] is zero.

Solution: Using determinant, compute: |B| = 3(x*3 - 20) - 1(1*3 - 8) + 2(5 - 2x) = 0. Solve the resulting equation for x.

6. Discuss the relationship between the determinants of a matrix and its inverse. What condition must the original matrix satisfy for its inverse to exist?

Solution: For a matrix A to have an inverse A^(-1), it must be non-singular; hence, det(A) ≠ 0. The relationship is |A^(-1)| = 1/det(A).

7. Calculate the determinant of the matrix C = [[sin(x), cos(x)], [cos(x), -sin(x)]] and interpret the result.

Solution: Det(C) = sin(x)(-sin(x)) - cos(x)(cos(x)) = -sin^2(x) - cos^2(x) = -1 (using Pythagorean identity). The result indicates that matrix C is singular.

8. Using Cramer's Rule, solve the system of equations represented by A = [[2, 3], [1, 2]], bx = [[7], [4]] for x.

Solution: Det(A) = 1. Use Cramer’s rule: x1 = |7, 3|/|2, 3|, x2 = |2, 7|/|2, 3|; x1 = (7*2 - 3*4) = 14 - 12 = 2, x2 = (2*7 - 3*1)/1 = 11. Therefore, x = (2, 3).

Answers

Check your answers with the solutions below.

9. Find the minor and cofactor of the element a23 in the matrix D = [[2, 3, 4], [5, 6, 7], [8, 9, 1]].

Solution: Minor M23 involves deleting row 2 and column 3: M23 = |[[2, 3], [8, 9]]| = (2*9 - 3*8) = 18 - 24 = -6. Cofactor A23 = (-1)^(2+3) * M23 = 6.

10. Explain why the area of a triangle formed by collinear points is zero. Use a determinant to illustrate your explanation.

Solution: If points are collinear, then the determinant of their coordinate matrix is zero, indicating no area. For example, |[[1, 1, 1], [1, 2, 2], [2, 3, 2]]| = 0.

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